Calculation Policy

for

Mathematics

Haydon Bridge Partnership

2014

Introduction

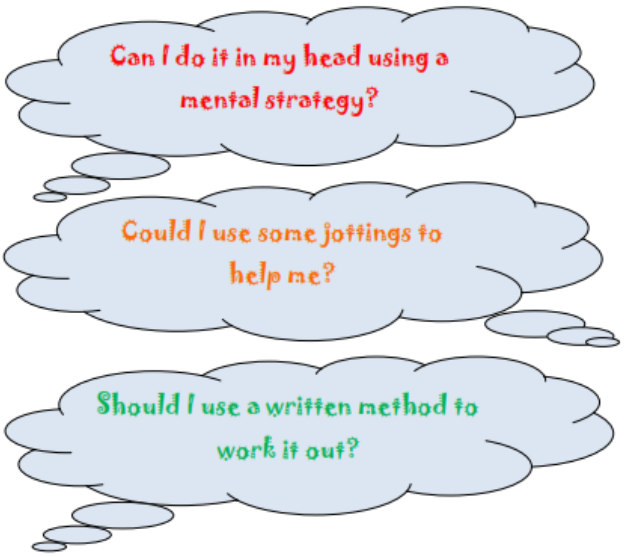
The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the Key Stage 1 and 2 phases. Please note that early learning in number and calculation in Reception follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014 and the expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.



Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved.

Approximate
Calculate
Check

ADDITION
Year 1

Add one-digit and two-digit numbers to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on. Consider very carefully the language used!

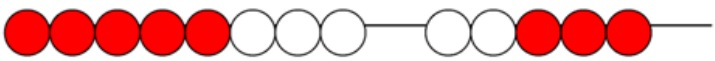
Children should:

- Have access to a wide range of counting equipment such as everyday objects, number tracks and number lines, Base 10 apparatus, arrow cards, bead strings etc., and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = ☐ 15 + 4 = ☐ 5 + 3 + 1 = ☐

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

8 + 5

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

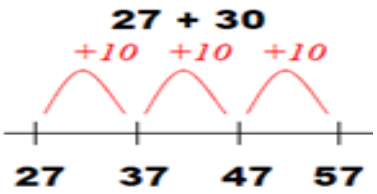
Key skills for addition at Y1:

- Count, read and write numbers to 100 in numerals, incl. 1—20 in words
- Represent and use number bonds and related subtraction facts within 20.
- Count to and across 100 boundary , forwards and backwards, beginning with 0 or 1, or from any given number.
- Count in multiples of twos, fives and tens.

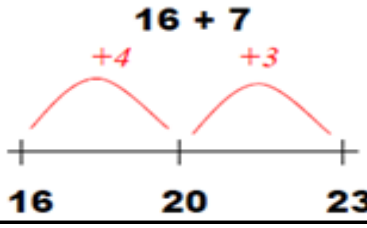
ADDITION
Year 2
Add with two-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

Add 2-digit numbers and tens:

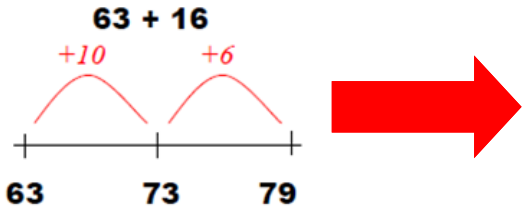


Add 2-digit numbers and units:



Use empty number lines, Base 10 apparatus, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the expanded column method when secure adding tens and units:



	2	3
+	1	7
	1	0
	3	0
	4	0

Add the units first in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

Model calculations with practical apparatus as much as possible.

To support understanding, pupils may physically make and carry out the calculation with Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

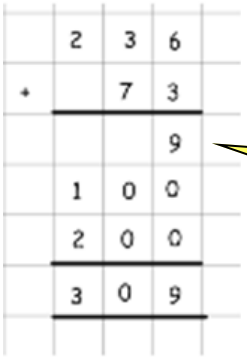
Key skills for addition at Y2:

- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law).
- Recall and use addition facts to 20 fluently, and derive and use related facts up to 100.
- Count in steps of 2, 3 and 5 and count in tens from any number, forward and backward.
- Recognise the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.
- Identify, represent and estimate numbers using different representations including the number line.

ADDITION
Year 3

Add numbers with up to 3-digits

Extend the **expanded column addition** method to HTU + TU:



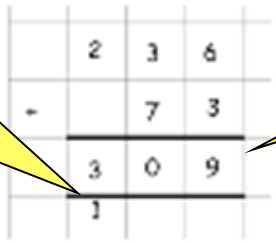
Add the units first in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

Move to the compact **column addition** method, with 'carrying'.

'Carry' numbers underneath the bottom line.



Add the units first.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind pupils that the actual value is '**three tens** add **seven tens**', not three add seven', which equals **ten tens**.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, 'carry', expanded, compact**

Key skills for addition at Y3:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.
- Count from 0 in multiples of 4, 8, 50. and 100.
- Compare and order numbers up to 1000.
- Find 10 or 100 more or less than a given number.

ADDITION
Year 4

Page 1 - Add numbers with up to 4 digits

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913

	3	5	1	7
+		3	9	6
	3	9	1	3
		1	1	

Add units first

'Carry' numbers **underneath** the bottom line.

What not to say:

7 + 6 = 13. Put 3 down and 1 on the doorstep.

Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

To begin with, reinforce correct place value by reminding them of actual values:

- 7 + 6 = 13. Put 3 in the units column and carry the 1 ten to the tens column.
- 1 tens + 9 tens + 1 ten = 11 tens. Put 1 ten in the tens column and carry 1 hundred to the hundreds column.
- 5 hundreds + 3 hundreds + 1 hundred = 9 hundreds etc.

Eventually, simplify language using "of these" (see page X)

Extend to adding 3 whole numbers e.g. 3517+ 396 + 43

Use and apply this method to money and measurement values.

ADDITION
Year 4

Page 2 - Add numbers with up to 4 digits

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, **thousands, hundreds, digits, inverse**

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

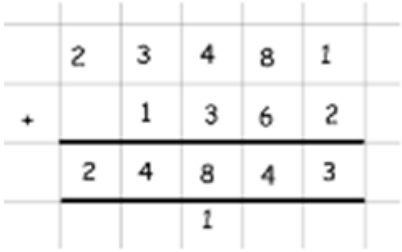
ADDITION
Year 5

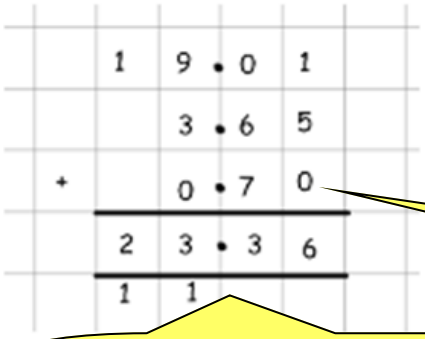
Add numbers with more than 4 digits

including money, measures and decimals with different numbers of decimal places.

The decimal point should be aligned in the same way as the other place value columns. Decimal points should be **on the dividing line** between the units and the tenths, not in a column of their own!

Numbers should exceed 4 digits.





Pupils should be able to add more than two values, carefully aligning place value columns.

Empty decimal places can be filled with zero to show the place value in each column.

Say '6 tenths add 7 tenths' to reinforce place value. Eventually, use the phrase "of these" to simplify language.

Children should: Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

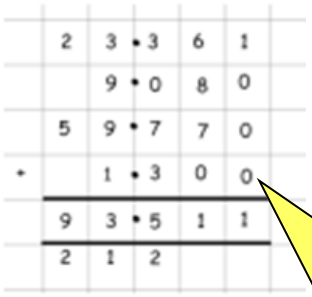
Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

ADDITION
Year 6

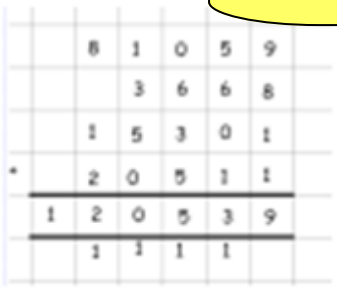
Add several numbers of increasing complexity



Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.

Empty decimal places can be filled with zero to show the place value in each column.



Adding several numbers with more than 4 digits.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

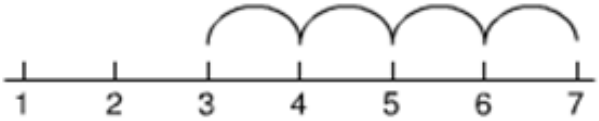
SUBTRACTION Year 1

Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20



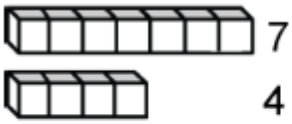
$7 - 4 = 3$

Remember always to reinforce the link between addition and subtraction.

Model subtraction using hundred squares and numbered number lines/tracks and practically.

Find the 'difference between'

This will be introduced practically with the language '**find the difference between**' and '**how many more?**' in a range of familiar contexts.



'Seven is 3 more than four'

'I am 2 years older than my sister'

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Y1:

- Given a number, say **one more or one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

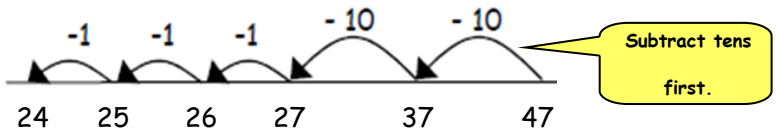
SUBTRACTION
Year 2
Subtract with 2 digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills. The strategy should be modelled using Base 10 apparatus.

This strategy will be used for: **2-digit numbers subtract units** (by taking away / counting back) e.g. 36—7
2-digit numbers subtract tens (by taking away / counting back) e.g. 48—30
Subtracting pairs of 2-digit numbers (see below:)

Subtracting pairs of 2-digit numbers on a number line:

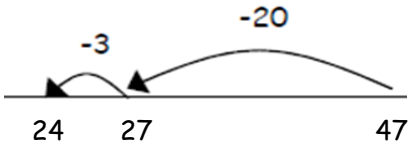
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



Subtract tens first.

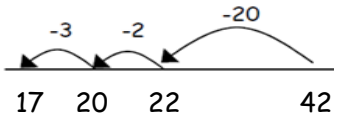
Then subtract units.

Move towards more efficient jumps back, as below:



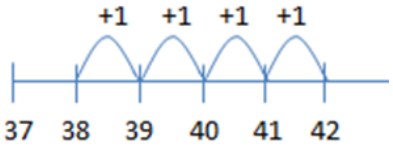
Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Mental strategy - subtract numbers close together by **counting on:**

42 – 38 = 4



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

They should also be encouraged to use more efficient ways to count on than counting in ones, using number bonds, e.g.

38 $\xrightarrow{+2}$ 40 $\xrightarrow{+7}$ 47

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? **difference, count on, strategy, partition, tens, units**

Key skills for subtraction at Y2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

SUBTRACTION
Year 3
Subtract with 2 and 3 digit numbers

STEP 1: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

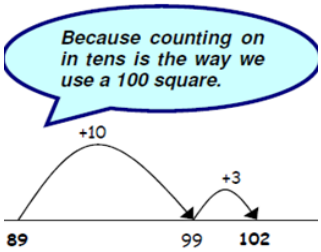
Column subtraction method

STEP 2: Once pupils are secure with the understanding of 'exchanging', they can use the column method to subtract any 2 then 3-digit numbers.

	$^{6}\cancel{7}$	$^{1}2$
-	4	7
	2	5

	$^{2}\cancel{3}$	$^{1}4$	6
-	1	8	2
	1	6	4

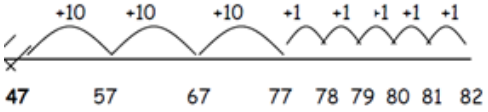
Use vocabulary 'change', not 'borrow'.



Counting on as a mental strategy for subtraction:

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

- Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:



Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Y3:

- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds** .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10
- Read and write numbers up to 1000 in numerals and words.
- Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

SUBTRACTION Year 4

Subtract with up to 4 digit numbers

Column subtraction method using larger numbers

As introduced in Y3, but moving towards larger numbers and money.

Language can be simplified using the phrase "of these".....

Change one of these ....

....into ten of these.

	2	$^{6}\cancel{7}$	$^{1}5$	4
-	1	5	6	2
	1	1	9	2

	£		
	$^{4}\cancel{5}$.	$^{1}1$	7
-	2 .	4	3
	2 .	7	4

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method. Counting on is particularly useful for finding the difference.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

SUBTRACTION Year 5

Subtract with at least 4 digit numbers

including money, measures, decimals.

Compact column subtraction

Subtracting with larger integers.

	$^{2}\cancel{3}$	$^{10}\cancel{1}$	$^{1}0$	$^{3}\cancel{4}$	$^{1}6$
-		2	1	2	8
	2	8	9	1	8

Including calculations which need several changes.

418.5 - 78.26

	$^{3}\cancel{4}$	^{11}	8 •	$^{4}\cancel{5}$	$^{1}0$
-		7	8 •	2	6
	3	4	0 •	2	4

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Add a 'zero' in any empty places to aid understanding of what to subtract in that column.

Create lots of opportunities for subtracting and finding differences with money and measures.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly large numbers .
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

SUBTRACTION
Year 6

Subtract with increasingly large and more complex numbers and decimal values

	6	$^{7}\cancel{8}$	$^{1}\cancel{0}^{9}$	$^{1}0$	$^{3}\cancel{4}$	$^{1}5$
-	4	2	3	1	3	8
	2	5	6	9	0	7

Using the compact column method to subtract more complex integers. Note the effect of the two zeros in the middle two columns.

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

5268 - 734.76

	$^{4}\cancel{5}$	$^{1}2$	6	$^{7}\cancel{8}$ •	$^{1}\cancel{0}^{9}$	$^{1}0$
-		7	3	4 •	7	6
	4		3	3 •	2	4

Empty places can be filled with **zeros** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

MULTIPLICATION
Year 1

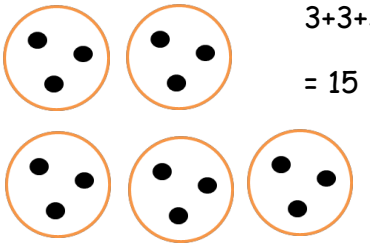
Multiply with concrete objects, arrays and pictorial representations

How many legs will 3 teddies have?

2 + 2 + 2 = 6

There are 3 sweets in one bag.

How many sweets are in 5 bags altogether?



3+3+3+3+3

= 15

Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Y1:

Count in multiples of 2, 5 and 10.

Solve one-step problems involving multiplication, by calculating the answer using concrete objects,

pictorial representations and arrays with the support of the teacher.

Make connections between arrays, number patterns, and counting in twos, fives and tens.

Begin to understand doubling using concrete objects and pictorial representations.

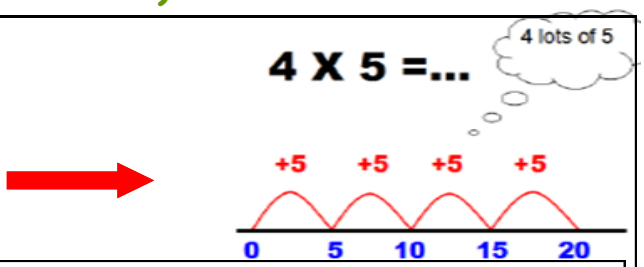
MULTIPLICATION
Year 2

Multiply using arrays and repeated addition

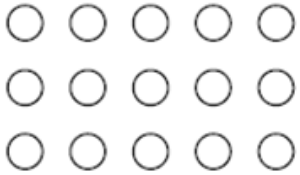
(using at least 2s, 5s and 10s)

Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



Use arrays:



5 x 3 = 15

3 x 5 = 15

4 X 5 = 20

3 x 5 = 5 + 5 + 5 = **15**

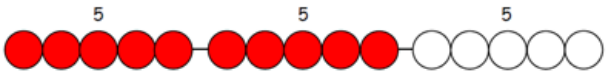
5 x 3 = 3 + 3 + 3 + 3 = **15**

Constantly reinforce the link between multiplication and division.

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as **3 x __ = 6**.

Use practical apparatus:

5 x 3 = 5 + 5 + 5



Use mental recall:

- Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Key skills for multiplication at Y2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the x and = signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

MULTIPLICATION
Year 3
Multiply 2-digits by a single digit number

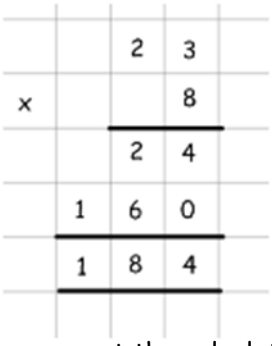
Introduce the grid method for multiplying 2-digit by single-digits:

Eg. 23 x 8 = 184

x	8
3	**2 4**
2 0	**1 6 0**

← Line up the digits in place value columns.

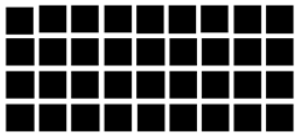
Then add in columns: **1 8 4** **(ensure the 8 isn't added)**

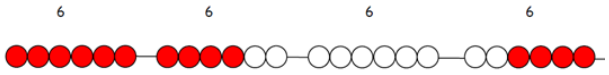


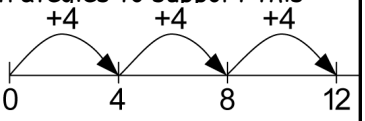
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

To do this, children must be able to: 3 **2 4**

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:







Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., **partition, grid method, multiple, product, tens, units, value**

Key skills for multiplication:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using commutativity (**4 × 12 × 5** = 4 × 5 × 12 = **20 × 12** = 240) and for missing number problems □ x 5 = 20, 3 x □ = 18, □ x □ = 32

MULTIPLICATION
Year 4

Multiply 2 and 3-digits by a single digit

Using all multiplication tables up to **12 x 12**

Developing the grid method: Eg. 136 x 5

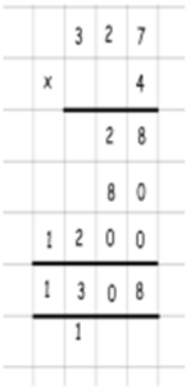
X	5
6	3 0
3 0	1 5 0
1 0 0	5 0 0

Line up the digits in place value columns.

Then add in columns: 6 8 0

Encourage estimating and checking to ensure accuracy.

Developing the expanded column method:



Move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in "carrying" for written addition.

Children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g: —**346 x 9** is approximately 350 x 10 = **3500**

Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables **up to 12 x 12**

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times, as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers.
- Use place value, known facts and derived facts to multiply mentally.

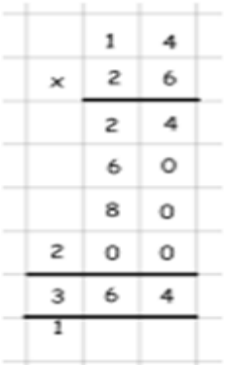
Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.

- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

MULTIPLICATION
Year 5

Multiply up to 4-digits by 1 or 2 digits

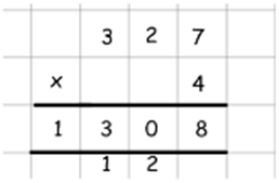
Extend expanded column multiplication to TU x TU



Introducing column multiplication

Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28



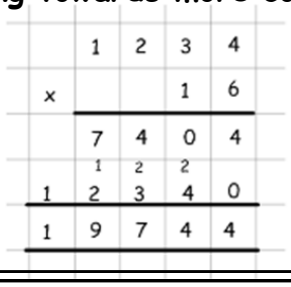
Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

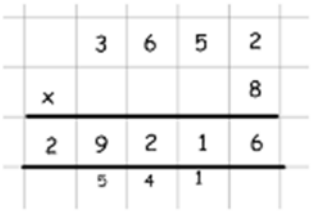
Introduce **long multiplication for multiplying by 2 digits**

x	10	8
10	100	80
3	30	24

18 x 3 on the 1st row (8 x 3 = 24, carrying the 2 for twenty, then „1" x 3). 18 x 10 on the 2nd row. Put a zero in units first, then say 8 x 1, and 1 x 1.

Moving towards more complex numbers:





Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, 'carry'**

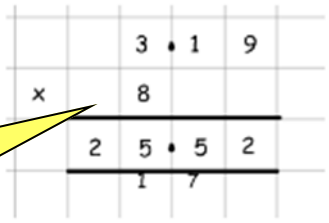
Key skills for multiplication at Y5:

Identify multiples and factors, using knowledge of **multiplication tables to 12x12.**

Solve problems where larger numbers are decomposed into their factors

MULTIPLICATION
Year 6

Short and long multiplication as in Y5, and multiply decimals with up to 2 d.p by a single digit



Remind children that the single digit belongs in the units column.

Line up the decimal points in the question and the answer.

This works well for multiplying money (£.p) and other measures.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit**.
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number**.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, „carry", **tenths, hundredths, decimal**

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5).**
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

DIVISION
Year 1

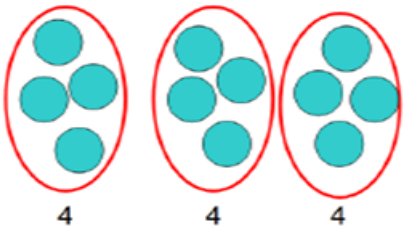
Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

How many groups of 4 can be made with 12 stars? = **3**

Grouping:

Sharing:



12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement… ?

"18 shared between 6 people gives you 3 each."

Pupils should :

- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between „grouping" objects (How many groups of 2 can you make?) and "sharing" (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Key Vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array

Key number skills needed for division at Y1:

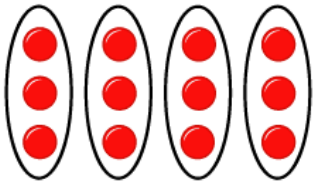
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

DIVISION
YEAR 2

Group and share small quantities

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

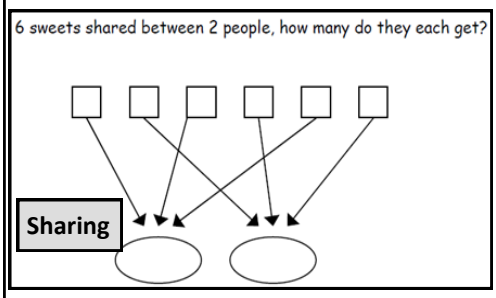
Arrays:

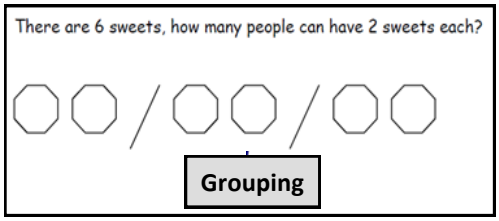


12 ÷ 3 = 4

This represents **12 ÷ 3, posed as** how many groups of 3 are in 12? Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing and grouping:





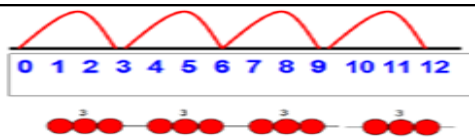
Important! The words "divide" and "divide by" and the symbol "÷" are ambiguous. For example, 28 ÷ 7 can mean two different things.

Note also that objects are "shared between", not "shared by".

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could and using a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?' **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Pose **12 ÷ 3** as '**How many groups of 3 are in 12?**'

When sharing, make connection between 12 shared between 3 and $\frac{1}{3}$ of 12.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Key number skills needed for division at Y2:

Cuisenaire and Thinking Blocks are useful resources for division.

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

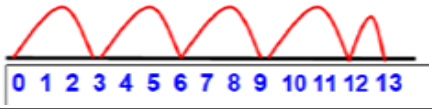
DIVISION
Year 3

Divide 2-digit numbers by a single digit

(where there is no remainder in the final answer)

Grouping on a number line:

13 ÷ 3 =



STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders,** as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

Short division:

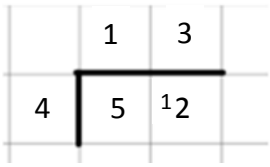
Limit numbers to **NO** remainders in the answer **OR** carried (each digit must be a multiple of the divisor).

STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all.

Note: The language used at this point is crucial to understanding. It is meaningless to say "How many threes in 9?" It is much better to say "If you share 9 tens between 3 people, how many do they get each?" (They get 3 tens each so write 3 in the tens column above the 9). This format may seem wordy by comparison, but it can later be shortened to: "Share 9 of these between 3 people." The phrase "of these" can be used in any column.

Short division:

Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the calculation.



STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96 ÷ 4), and be taught to 'carry' the remainder onto the next digit, still using appropriate language, e.g. "Change 1 of these into 10 of these."

If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, **inverse, short division, 'carry', remainder, multiple**

Key number skills needed for division at Y3:

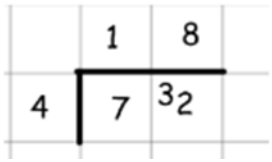
- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, Connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and

DIVISION
Year 4

Divide up to 3-digit numbers by a single digit

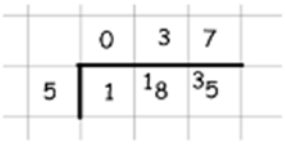
(without remainders initially)

Continue to develop short division:



STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder**—see steps and appropriate language in Y3), but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process (see example).

STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit. However problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Y5 level.



When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the unused number over to the next digit as a remainder.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, **divisible by, factor**

Key number skills needed for division at Y4:

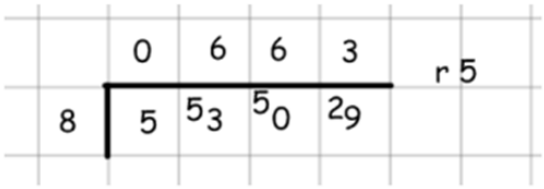
- **Recall multiplication and division facts for all numbers up to 12 x 12.**
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

DIVISION
Year 5

Divide up to 4 digits by a single digit

Short division, including remainder answers:

including those with remainders.



Include money and measure contexts.

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, **pupils need to consider the meaning of any remainder** depending upon the context of the problem. Examples:

1) Share 25 marbles between 4 children: 25 ÷ 4 = 6 r 1

2) 25 people are asked to make groups of 4. How many groups of 4 can be made? 25 ÷ 4 = 6 r 1 ⟶ 6 groups

3) 25 people need to be taken by taxi to a theatre. Each taxi holds 4 passengers. How many taxis are needed?

25 ÷ 4 = 6 full taxis, remainder 1 person ⟶ 7 taxis.

If children are confident and accurate:

- Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation—see Year 6.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

Key number skills needed for division at Y5:

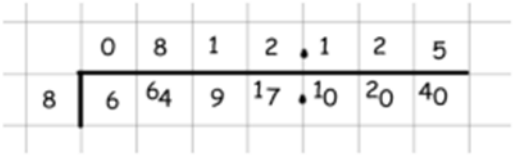
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g.98÷4=24r2=24½= 24.5 ~ 25).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

DIVISION
Year 6

Divide at least 4 digits by both single-digit and 2-digit numbers

(including decimal numbers and quantities)

Short division, for dividing by a single digit: e.g. 6497 ÷ 8



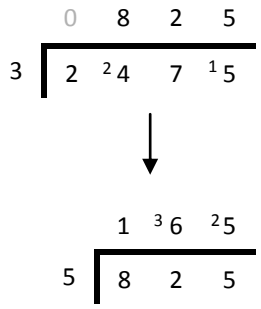
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved. E.g. Divide £25 between 4 people: 25 ÷ 4 ⟶ 25.00 ÷ 4 = 6.25 ⟶ £6.25

Dividing by 2 digits.

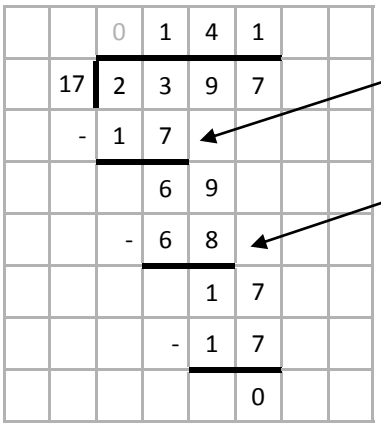
Method 1: Where the divisor can be factorised

Consider the calculation 2475 ÷ 15. Now consider the divisor as 3 x 5. Therefore the calculation can be thought of as 2475 divided by 3 and by 5.



Method 2: Where the divisor can't be factorised

Consider the calculation 2227 ÷ 17. The divisor cannot be factorised (other than 1 x 17), so there is no alternative but to divide by 17.



1 x 17 = 17. Subtract from 23 to give 6. Then bring down the 9 to join the 6.

4 x 17 = 68 Subtract from 69 to give 1. Then bring down the 7 to join the 1, etc.

Note that the above language does not reinforce the proper values of the numbers, but that there is little alternative!

Key Vocabulary: As previously, & common factor

Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.